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FINITE-AMPLITUDE WAVES IN A STRATIFIED JET STREAM AND CLEAR AIR TURBULENCE

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16. Abstract

The nonlinear problem of stability of internal waves in a stratified jet stream is solved by the amplitude power expansion method in accordance with the "two-time method". The conclusion is reached that in a free atmosphere hydrodynamically unstable layers become turbulent very rapidly; they may practically always be considered to be turbulent layers. It is shown that the layer of the tropopause may be regarded as a particular layer which has the greatest probability of containing zones of "secondary instability" and turbulent patches, which are formed by internal gravitational shear waves of finite amplitudes.

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Z. N. Kogan and N. P. Shakina

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Based on present concepts [1, 2], the instability of wave perturbations, their amplitude increase, and disintegration are the basic mechanism forming the zones of intense turbulence in a free atmosphere. Recently the linear problem of the stability of streams which are continuously stratified in terms of wind and temperature has been intensively studied [3-6]. It has been established that at Ri numbers which are smaller than 1/4, the perturbations of infinitely small amplitude lose stability in a very wide range of wavelengths; this range of wavelengths contracts as the minimum Ri number increases in the stream. Laboratory experiments [7] have shown that at the beginning stage of the perturbation increase (i.e., at fairly small amplitudes) the linear theory closely describes the development of instability. A theoretical study of a further increase in unstable waves requires an examination of perturbations of finite amplitude. One of the fruitful approaches to a solution of non-linear problems of stability is based on the concept of L. D. Landau [8], which represents the characteristics of perturbed motion in the form of amplitude power series. The terms of first order in these series correspond to the linear theory, whose higher terms give nonlinear corrections. In combination with the so-called "two time method", the amplitude power expansion method was used in [9] to analyze the stability of the flow of a

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viscous homogeneous liquid (Poiseuille and Couette-Poiseuille flow). The procedure developed in [9] was used in the study for the case of a stratified medium to investigate "super-critical" (corresponding to Ri numbers which are less than the critical value of 1/4) behavior of waves which are unstable according to linear theory, and also internal gravitational shear neutral waves of finite amplitude for all Ri>0. We shall briefly discuss a method to solve this problem.

Let us assume the x axis is directed along the flow (along the axis of the plane-parallel stream), z— is directed upward (all the variables are dimensionless). We shall solve the non-linear system of equations consisting of the equations of motion, heat flux, and continuity in the Boussinesq approximation under conditions (which are normal for nonviscous problems of hydro-dynamic stability) when there is no flow on the layer boundaries. /334

We should recall that in linear theory perturbations are usually represented in the form of elementary waves of the form

$$s'=\hat{s}(z)\exp\left(i(\alpha x+\beta y+\omega^{\circ} t)\right),$$

where s' is a fluctuation of any of the values; s— the corresponding amplitude function; α and β— wave numbers along the x and y axes; t— time; $\omega^{\circ}=\omega_r+i\omega_i$ — complex frequency. The exponential factor $A\sim\exp(-\omega_i t)$ in the case $\omega_i<0$ describes the initial increase in the perturbation amplitude with time.

Returning to the nonlinear problem, let us replace the variables

$$r=\alpha x+\beta y+\omega_r t,\quad \omega_r=\omega(A),\quad A=A(t),\quad z=z. \qquad (1)$$

Let us also introduce the "current function" by the relationships

$$\frac{\partial\psi}{\partial z}=\alpha u+\beta v, \quad \frac{\partial\psi}{\partial r}=-w \tag{2}$$

where u, v, w — are the components of the perturbed velocity. We shall try to determine the current function ψ and the deviation θ' of the potential temperature from its initial unperturbed value in the form of the sum of the basic wave and its harmonics (the tilde designates the complex conjugate)

$$\psi=\psi^{(k)}(A, z)e^{ikr}+\tilde{\psi}^{(k)}(A, z)e^{ikr}. \tag{3}$$

The terms with k=0 describe the nonperiodic "average" motion (basic flow and "secondary flow"); k=1— harmonics of the "primary wave". Terms with k>1 are the result of the interaction of the lowest terms of the series. Following [9], we shall also assume that

$$\psi^{(k)}(A, z)=\psi^{(k, n)}(z)A^n, \tag{4}$$

and it is assumed that the "amplitude parameter" A is finite, but sufficiently small so that the series (4), may converge, in which it is necessary that $n\geqslant k$. This condition actually means that the k^{th} harmonics, which arises during the nonlinear interaction, will be of the k^{th} order of smallness with respect to A [in the expansion (4) only terms beginning with A^k are present]. This is a key condition and will make it possible to "split" the system of equations. The real function of time A(t) again describes the amplitude change with time.

Another basic assumption is as follows [9]

$$\begin{aligned} A^{-1}\frac{dA}{dt}&=a^{(0)}+a^{(1)}A+a^{(2)}A^2+\ldots=a^{(n)}A^n, \\ \omega+\frac{d\omega}{dA}\left(t\frac{dA}{dt}\right)&=b^{(0)}+b^{(1)}A+b^{(2)}A^2+\ldots=b^{(n)}A^n. \end{aligned} \tag{5}$$

Here $a^{(n)}$, $b^{(n)}$ are unknown constants. In the assumptions (3) - (5), the initial nonlinear system is reduced to two groups (which are infinite with respect to the indices n and k) of ordinary differential equations with respect to $\psi^{(k,n)}$ и $\theta^{(k,n)}$, which may be solved sequentially beginning with n = k = 1. Turning to the higher terms n, k, it is first necessary to determine $a^{(n)}$ and $b^{(n)}$ in (5). Assuming k = n = 1, we obtain an approximation of the order O(A), which completely coincides with the linear stability problems. Thus $a^{(o)}$, $b^{(o)}$ are determined as eigenvalues of this linear problem: $a^{(0)}=-\varkappa c_i^{(0)}$, $b^{(0)}=-\varkappa c_r^{(0)}$, where $c_r+ic_i=c$ is the phase velocity of the primary wave $\varkappa^2=\alpha^2+\beta^2$. The coefficients $a^{(n)}$, $b^{(n)}$ with even indices equal zero [8,9]; with odd indices they are determined as follows. /335

The differential equations with respect to $\psi^{(k,n)}$ have the form

$$L_n\psi^{(k,n)}=\lambda f-g, \tag{6}$$

where the unknown coefficients $a^{(n)}$, $b^{(n)}$, play the role of the parameter λ. The operator L_n is linear, and f and g are known functions which include the lower terms of the expansion (4). The boundary conditions for (6) are homogeneous. Let us use $\chi^{(k,n)}$ to designate the solution of the problem which is conjugate to the homogeneous problem (6). The solution $\psi^{(k,n)}$ will only exist if the homogeneous problem (6) has eigen solutions (for certain so-called eigenvalues λ. Multiplying both parts of (6) by χ and integrating with respect to z within the limits of the layer (z_1, z_2) being studied, we obtain the following relationship for determining λ.

$$\lambda=\int_{z_1}^{z_2} g\chi dz \Big/ \int_{z_1}^{z_2} f\chi dz. \tag{7}$$

The parameter λ may assume as many values as there are solutions of the problem of eigenvalues for $L_n\psi^{(k,n)}=0$, in the case of the previous homogeneous boundary conditions.

We have investigated terms in the expansions of (3) - (5) up to the second harmonic and third order inclusively, i.e., values of $a^{(2)}$, $b^{(2)}$ and the functions $\psi^{(0,2)}, \theta^{(0,2)}$ (which describe secondary flow), $\psi^{(1,1)}, \theta^{(1,1)}$ (harmonics of a "primary" basic wave provided by linear analysis), $\psi^{(1,3)}, \theta^{(1,3)}$ (distortion of the basic wave due to nonlinear effects), $\psi^{(2,2)}, \theta^{(2,2)}$ (second harmonics which is double the frequency), with respect to the stabilities $a^{(0)}, b^{(0)}, \psi^{(1,1)}$ and $\theta^{(1,1)}$. found from the solution of the linear problem. For example, let us write the equations by means of which $\psi^{(1,1)}$ and $\psi^{(2,2)}$ were found.

$$D^2\psi^{(1,1)}-\psi^{(1,1)}\left[\frac{D^2\hat{u}}{\hat{u}-c}-\frac{N^2}{(\hat{u}-c)^2}+\frac{H^2}{L^2}\varkappa^2\right]=0. \qquad (8)$$

Here $\hat{u}(z)=\bar{u}(z)\cos\varphi;\ \varphi=\operatorname{arctg}\alpha/\beta;\ \bar{u}(z)$ is the velocity of the unperturbed flow, whose profile was selected as jet-like in the calculations, $\bar{u}=Ue^{-z^2}$; $N^2=g(\gamma_a-\gamma)H^2/\theta U^2$ — dimensionless frequency of Brent-Weissel (the Ri number which is characteristic for the entire flow is of importance); H,L — vertical and horizontal scales of the perturbations, respectively: $H\sim1\div2$ км, $L\sim10$ км, $U\sim10$ m/sec. These characteristic values may be used for changing to the dimensional values; $\gamma=-d\bar{T}/dz$, γ_a — dry adiabatic gradient; D=d/dz. For $\psi^{(2,2)}$, we have

$$D^2\psi^{(2,2)}-\psi^{(2,2)}\left[\frac{D^2\hat{u}}{\hat{u}-c}-\frac{N^2}{(\hat{u}-c)^2}+\frac{4H^2}{L^2}\varkappa^2\right]=$$
$$=\frac{[\psi^{(1,1)}]^2}{2\varkappa(\hat{u}-c)^2}\left[\frac{3N^2D\hat{u}}{(\hat{u}-c)^2}+\frac{2N_1^2-D^2\hat{u}D\hat{u}}{\hat{u}-c}+D^3\hat{u}\right]. \qquad (9)$$

Here $N_1^2=gH^2D\gamma/TU^2$; equations (8), (9), which are more cumbersome and therefore the remaining equations are not derived here, may be numerically integrated on the Minsk-22 and Minsk-32 computers.

The dependence of the parameter A on time, with allowance for the terms of second order in (5), may be described by the following equation

$$A(t)=\left[\frac{a^{(0)}}{De^{-2a^{(0)}t}-a^{(2)}}\right]^{1/2}, \qquad (10)$$

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where D is a constant which we may determine assuming that at the moment $t=t_0$ the perturbation amplitude is known

$$A(t_0)=A_0, \quad D=a^{(0)}A_0^{-2}+a^{(2)}. \qquad (11)$$

The wave frequency also depends on the amplitude (and on time)

$$\omega(t)=b^{(0)}-\frac{a^{(0)}b^{(2)}}{a^{(2)}}+\frac{1}{t}\frac{b^{(2)}}{a^{(2)}}\ln\frac{A}{A_0}. \qquad (12)$$

This latter dependence is weakly expressed since $\ln(A/A_0)$ is an almost linear function of time. The frequency changes become significant only when there is a rapid growth in the amplitude of the unstable wave.

As may be seen from (10), at $a^{(2)}>0$ the perturbation increase rate is greater than that provided by linear theory: there is a regime of "rigid perturbation" [10] or a "supercritical instability". The presence of a discontinuity A(t) when the denominator in (10) vanishes has an influence upon the limited nature of the investigation, which makes it impossible to describe the increase in the wave for all t. Certain physical concepts regarding the length of the time interval in which the model formulated is valid will be given below. If $a^{(2)}<0$, then A(t) strives to a finite limit which equals $A^{\circ}=(-a^{(0)}/a^{(2)})^{1/2}$. This is a case when the instability leads to the occurrence of periodic motion: "a regime of soft excitation" [10].

The calculations show that in a jet stream with a profile of unperturbed velocity $\bar{u}=U\exp(-\mu z^2)$, $\mu>0$ and a linear temperature profile $\bar{T}(z)$

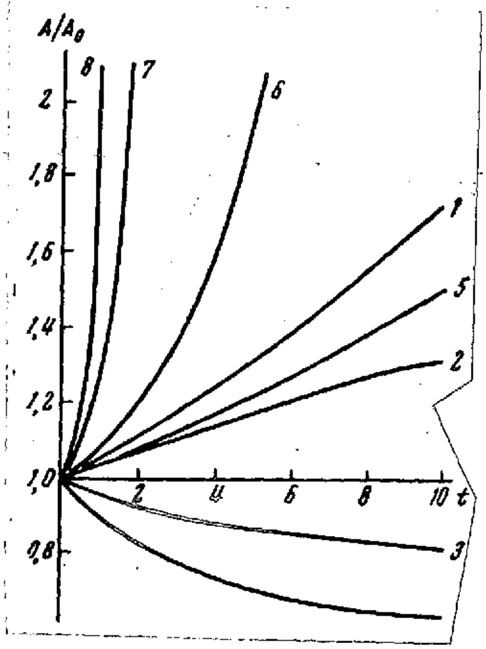

Figure 1. Amplitude of the perturbation A(t) for cases of periodic motion or a "soft excitation regime (curves (1-4) and a rapid amplitude increase, or "rigid excitation" regime (curves 5-8) at infinitely small A_0 (curves 1 and 5), at $A_0=3\cdot10^{-3}$ (curves 2 and 6), at $A_0=7\cdot10^{-3}$ (curves 3 and 7) and at $A_0=10^{-2}$ (curves 4 and 8).

the waves (which are unstable according to linear theory and were described in [6]) in the case of finite amplitudes may lead to the formation of supercritical regimes both of soft and rigid excitation. (The dependence of the supercritical regime on Ri and the wave number for the entire instability region must still be determined; our results only refer to several points in this region). The rate at which A(t) changes depends on the initial amplitude A_0. Figure 1 shows graphs of A(t) for two waves which are unstable in the linear approximation, but for one of which the rigid excitation $(N^2=1{,}8\cdot10^{-6}$ sec$^{-2})$ is found, and for the other the soft excitation (periodic regime) and $(N^2=1{,}2\cdot10^{-6}$ sec$^{-2})$. In both cases, the absolute value of $a^{(2)}$ is very large (on the order of 10^3). The limit to which the wave amplitude strives in the case of periodic motion is a value on the order of $10^{-2} - 10^{-3}$

in all the cases examined. Thus A(t) approaches A* from above if $A_0 > A^*$ (curves 3 and 4 in Figure 1 to which $A^* = 5{,}3 \cdot 10^{-3}$ corresponds), and from below if $A_0 < A^*$ (curve 2). Waves, whose amplitude increases are of the greatest interest in terms of the physics of the process. Let us examine in more detail the example of rigid excitation shown in Figure 1. /337

Let us give the parameters (dimensionless) of the primary wave: $\alpha=5$, $\beta=1$, $a^{(0)}=0{,}04$ at $N^2=1{,}8\cdot 10^{-4}$ sec^{-2}, where $N^2=g(\gamma_a-\gamma)/\theta$ is the Brent-Weissel frequency. We shall use $3 \cdot 10^{-3}$ as A_0. The periodic portion of the perturbed motion, within an accuracy of terms on the order A^3, i.e., the function

$$\psi' = A\psi^{(1,1)}e^{i\tau} + A^2\psi^{(2,2)}e^{2i\tau} + A^3\psi^{(1,3)}e^{i\tau}$$

will be calculated as a function of the variables t, $r=r(x, y, t)$, z. Figure 2 gives a graph of its dependence on time when x=y=0. The overall increase in the amplitude without distortion of the wave form may be seen: the unimportant role of the secondary harmonics is characteristic in general for waves in a layer with a constant temperature gradient. At the time t=7 (which corresponds to 1 1/2 - 2 hours from the time that the perturbation increase begins in the case of a characteristic time on the order of 15 minutes) the maximum fluctuations of the horizontal u' and vertical w' components reach values on the order of 0.10 and 0.04 of the characteristic wind velocity U in an unperturbed flow. The stage of the wave crest reversal occurs along with the formation of vortices. An analysis of the field of the perturbed potential temperature provides a concept of the onset of the stage. In our problem, the temperature is a value which can be conserved, so that the isolines θ=const in the r, z plane may be regarded as a cross section by this plane of physical surfaces. The occurrence of closed isolines θ in the r,z plane, and a zone of convective instability along with them, indicates the beginning of the collapse of the wave (Figure 3). Intense mixing must begin

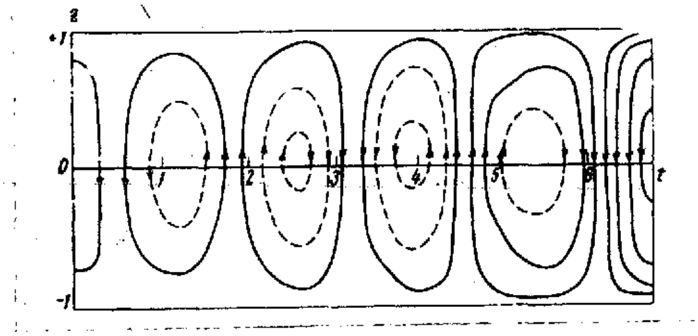

Figure 2. $\psi(z,t)$ isolines for the case of rigid excitation.

in these zones; smaller scale motions will develop, to which a portion of the primary wave energy will be changed. Motions within the $\partial\theta/\partial z<0$, zone, having much smaller (at least in order of magnitude) scales cannot be described within the framework of this model. The time interval for the applicability of the latter must probably be limited by the moment at which negative values of $\partial\theta/\partial z$ occur in the flow, i.e., the moment at which the wave collapse begins.

The greater is the initial amplitude of A_0 the more rapidly does the collapse begin. Thus, at $A_0=10^{-2}$ the closed isolines θ occur at the time t=0.8, i.e., 10-15 minutes after the beginning of the wave increase. The altitude distribution of the amplitudes and their value at the time of the reversal do not depend on A_0.

The increasing wave derives energy from the basic flow and transfers the momentum, deforming the velocity profile. We may /338 determine the exchange of energy between the main flow and the perturbations by examining the balance of nonperiodic flow energy. We may obtain an equation for the energy balance in the perturbed flow by the regular methods from the equations of motion

$$\frac{\partial E}{\partial t}+\psi_z\frac{\partial E}{\partial r}-\psi_r\frac{\partial E}{\partial z}=\frac{1}{\rho}\left\{(\alpha^2+\beta^2)\psi_z\frac{\partial p'}{\partial r}-\psi_r\frac{\partial p'}{\partial z}\right\}-\psi_r\frac{g\theta'}{\theta}.$$

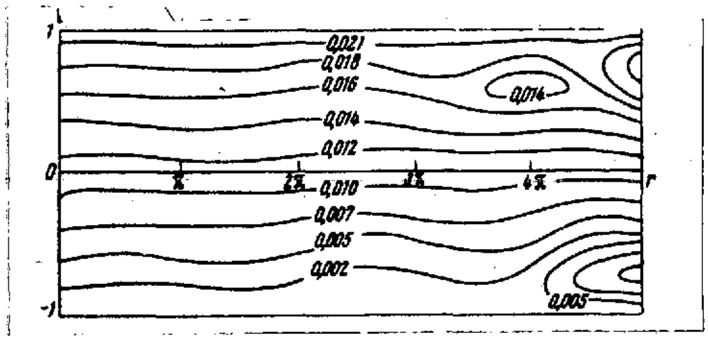

Figure 3. Isolines of perturbed temperature in the (r,z) plane for the case of rigid excitation; values above θ for z = -1 are given in degrees.

Here $E={}^1/_2(\psi_z^2+\psi_r^2)$ is the kinetic energy of the perturbed flow; p', θ' — deviations of pressure and temperature from their values at the initial moment. Substituting E in the form of a series with respect to the harmonic components (3), for $E^{(0)}$ — the energy of nonperiodic motion (i.e., the "average flow") — we obtain the equation for the energy balance per unit of mass.

$$\frac{\partial E^{(0)}}{\partial t}+\delta E_r^{(0)}+\delta E_D^{(0)}=\delta E_{P}^{(0)}+\delta E_{F}^{(0)}, \qquad (13)$$

where the energy transfer in the direction of the r axis and along the vertical is

$$\delta E_r^{(0)}=im(D\tilde{\psi}^{(m)}E^{(m)}-D\psi^{(m)}\tilde{E}^{(m)}), \quad 1\leqslant m<\infty$$

and

$$\delta E_z^{(0)}=im(\tilde{\psi}^{(m)}DE^{(m)}-\psi^{(m)}D\tilde{E}^{(m)}), \quad 1\leqslant m<\infty,$$

force of pressure gradient per unit time

$$\delta E_{P}^{(0)}=\frac{im}{\bar{\rho}}\varkappa^2[D\psi^{(m)}\tilde{p}^{(m)}-D\tilde{\psi}^{(m)}p^{(m)}], \quad 1\leqslant m<\infty,$$

buoyancy force per unit time

$$\delta E_{F}^{(0)}=\frac{img}{\bar{\theta}}[\tilde{\psi}^{(m)}\theta^{(m)}-\psi^{(m)}\tilde{\theta}^{(m)}], \quad 1\leqslant m<\infty.$$

Equation (13), which is integrated over altitude, represents the exchange of energy between perturbations and the "average flow" for a column of air with a unit base and a height equalling the thickness of the jet stream layer. We calculated the terms of the energy balance equation (13) on the order of A^2 and their integrals with respect to altitude. If the wave is neutrally stable ($a^{(0)} = 0$), then there is no exchange of energy between it and the basic flow. If $a^{(0)} \neq 0$, then the energy of the unperturbed motion may be transferred directly to the perturbations. In addition, there is a secondary flow, whose velocity is proportional to $D\psi^{(0,2)}$. /339

When the wave is close to reversal, the velocity of the secondary flow reaches 1 - 2% of the unperturbed flow velocity. The upper half of the flow close to the axis is somewhat accelerated, while the lower half is slowed down. The velocity redistribution in the main flow takes place as a result of vertical transport. The periodically distributed vertical velocities lead to organized outflow of the kinetic energy from the lower half of the jet stream to the upper half.

The order of the terms $\partial E^{(0)}/\partial t$, $\delta E_D^{(0)}$ and $\delta E_P^{(0)}$ is the same in Equation (13) and in essence they form the energy balance. Within an accuracy of terms on the order of A^2, there is no horizontal energy transport; the primary wave does not transfer energy in the horizontal direction. The buoyancy force makes a very small contribution to the energy balance, since the stability of the stratification in the hydrodynamically unstable layer is very small.

Assuming $A_0 = 3 \cdot 10^{-3}$ or $A_0 = 10^{-2}$ in the calculations, we find that at the initial moment the perturbations of the horizontal velocity component do not exceed 0.3 or 1% of the velocity of the basic flow. Such weak perturbations are practically always present in "quasi-laminar" flows in a free atmosphere. It may therefore be assumed that hydrodynamically unstable layers become turbulent immediately after it occurs and that the meso-scale motions in such layers are practically always turbulent. A clearly expressed turbulent zone may appear in the region $Ri < 1/4$, which arises under the influence of processes of a synoptic scale or any local processes — for example, the propagation of a long, internal wave with significant amplitude. Such a wave, deforming the profiles of temperature and wind in a hydrodynamically stable flow, produces zones of supercritical Ri which move together with the wave. A similar mechanism of "secondary instability", proposed by Phillips [11] may take place at comparatively small subcritical Ri.

Gravitational shear waves of finite amplitude, even though they may be neutrally stable, may be the reason for the occurrence of zones of instability and turbulent spots in the flow. A study of the nonlinear behavior of such waves (for which $a^{(0)}=0$) was performed within the framework of the model presented for different forms of the temperature profile in a jet stream. Except for the case $\gamma = -dT/dz = \mathrm{const}$ the distribution of $T(z)$, reproducing the temperature profiles in a layer of the tropopause of different types was examined: isothermal or polar type, $\gamma(z) = 0{,}0035(1-\mathrm{th}\,10z)$; inversion or tropical type $\gamma(z) = -0{,}007\,\mathrm{th}\,10z$; inversion-isothermal type $\gamma(z) = 0{,}0035(1-\mathrm{th}\,10z) - 0{,}007\exp\{-10(z-0{,}25)^2\}$; weak inversion type $\gamma(z) = 0{,}003 - 0{,}005\,\mathrm{th}\,10z$. The basic result of the calculations is as follows: neutral waves in a polytropic layer are practically linear even at very large amplitudes, whereas waves in a layer of the tropopause are nonlinear even for very small amplitudes. The behavior of the waves is determined always by the temperature profile; the wind distribution plays a secondary role.

Let us discuss in greater detail other important results for neutral gravitational shear waves of finite amplitude.

Waves which are neutrally stable according to linear theory remain neutrally stable always $(a^{(2)}=0)$. A change in the phase velocity described by the coefficient $b^{(2)}$ in a polytropic jet is always small, although it increases in terms of modulus with a decrease in stability. If the temperature profile has significant curvature (tropopause), the values of $b^{(2)}$ increase greatly, particularly in the first mode. The amplitude functions $\psi^{(i,3)}$ are increased — the additions to the amplitude function of the primary wave caused by a change in the wave phase velocity and the interaction of the first and second harmonics. The second harmonics $\psi^{(2,2)}$ also has a little greater influence in the case of a nonlinear profile $\bar{T}(z)$, as a result of which the wave form is distorted. The great influence of the temperature profile curvature is caused primarily by characteristics of the amplitude function $\psi^{(1,1)}$ of the primary wave, whose form always greatly depends on the profile $\bar{T}(z)$. The study [12] provides a detailed description of results of a numerical solution of the linear problem regarding internal waves, including a tropopause of different types. The presence of the tropopause has a slight influence on the wave phase velocities, but has a great influence upon the amplitude functions, so that the maximum of the wave activity is displaced toward the upper, more stable portion of the layer. In the lower portion of the layer (under the tropopause) the waves are suppressed more strongly, the smaller is the static stability. Thus, the amplitude of the primary wave is distributed nonuniformally in terms of altitude, particularly close to the tropopause level. As a result, there is an increase in the contribution of nonlinear terms which describe the vertical and horizontal transport of the perturbed values. If the stability is reduced in the lower portion of the layer /340 (under the tropopause), then even only moderately strong perturbations, which are nonuniformally distributed in terms of altitude, readily lead to the formation of instability regions (convective or hydrodynamic), that is, to the collapse of the wave crest under the tropopause level.

As an example, let us examine eigenoscillations of finite amplitude in a jet stream, whose axis coincides with the tropopause of the inversion type with a very thin (about 200 m) transition layer from the tropospheric gradient (0.7°/100 m) to the stratospheric gradient (-0.7°/100 m): $\gamma = -0{,}007\,\mathrm{th}\,10z$ where z — км, γ — deg/m.

If the perturbation amplitude is small $(A \leqslant 10^{-3})$, then the waves of any length which move along the flow in the range from 0.5 - 1 km to 20-40 km differ little from those described by linear theory. When the parameter A equals 0.05 - 0.1, then at first (for small A) the longest waves (20-30 km) and the shortest waves (1-2 km) and then the waves with a length of 5-10 km, begin to be deformed. As a result of the application of an increasing second harmonic, the wave crest becomes narrower, and the trough becomes more planar. The periodic portion of the current function $\psi(r,z)$ for this case equals the sum of the harmonic terms of the series (3) up to third order with respect to A inclusively, and is given in Figures 4 and 5 for two values of the parameter A. It may be seen that, with an increase in A, the form of the "stream cells" changes in the plane r,z. The closed stream lines in this plane are characteristic for traveling waves and do not coincide with the particle trajectories, reflecting only the instantaneous velocity distribution of the periodic portion of the perturbations. Their nonperiodic portion is secondary flow caused by the wave motion and directed along the main flow in the vicinity of the jet stream axis and against the flow on the periphery.

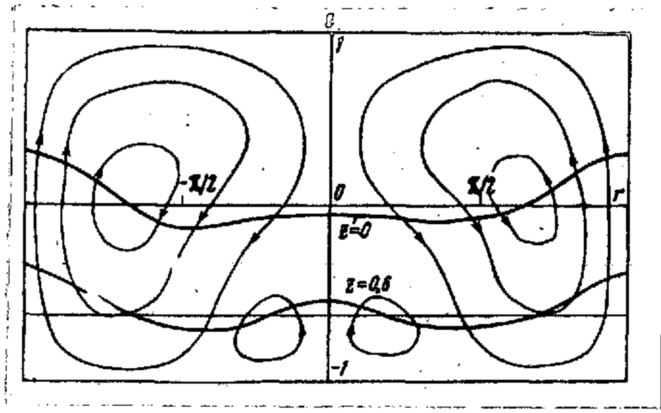

Figure 4. Current lines (thin lines) of wave motion in the (R,z) plane and the graph w(r) at z = 0 and z = -0.6 (heavy lines). for the case γ=-0.007 th 10z at A_0=0.01 (case of a neutrally stable wave).

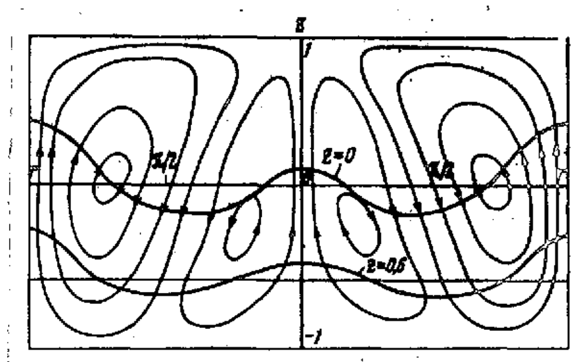

Figure 5. Current lines of wave motion and the graph w(r) at A_0=0.1 (case of neutrally stable waves).

In order to develop a concept regarding the form of the traveling waves, just as in the case of unstable waves, let us examine the isolines of a differing potential temperature in the r,z plane, (Figure 6). The closed isotherms (and the zones of convective instability) under the tropopause are clearly expressed for A=0.15. In the more stable portion of the layer — above the tropopause — wave-like oscillations of surfaces of differing potential temperature continue to exist. Thus, the neutral wave in the tropopause layer has a strong tendency to form "vortices" and a tendency toward reversal in the inversion portion /341 of the layer. Strong mixing must develop in the reversal zones in accordance with the mechanism of "secondary instability" (convective or hydrodynamic). For the implementation of this mechanism, the wave amplitude must reach a definite "critical" value, in our case close to A=0.12 ("the critical amplitude" is determined from the value $A=A_i$, at which those points appear in the flow where $\partial\theta/\partial z=0$). The concept of the "critical amplitude" was introduced by Phillips [13] in the problem of the energy spectrum of internal waves. (It was assumed that all of the waves reach critical amplitudes if turbulent spots are disseminated in the flow). This concept was developed further in [14].

The values of the "critical amplitudes", obtained in our calculations for jet streams in the tropopause (0.10 - 0.15), do not contradict the empirical data. The perturbations of the horizontal velocity component, which equal 10-15% of the main flow velocity, are frequently on the order of 10 km in atmospheric jet streams under conditions of stable stratification [15]. The values of the critical amplitudes determined, as has already been emphasized, by the form of the temperature profile, depend (to a lesser extent) also on the wind profile: according to the calculations, the contribution of nonlinearity is much greater (which means that the critical amplitude is smaller) in more intense and narrow jet streams. In addition, for given

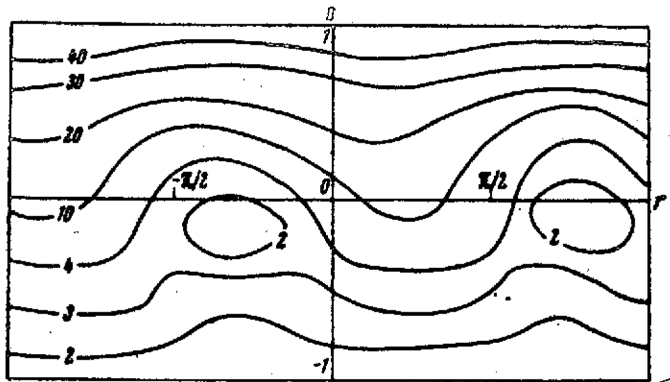

Figure 6. Isolines of potential temperature in the (r,z) plane (values of the temperature above its value on the lower layer boundary are given) for a jet stream with the axis on the tropopause level $(\gamma = -0{,}007\ \mathrm{th}\ 10z,\ c_0 = c_1 +)$ at $A_0 = 0.15$ (case of neutrally stable wave).

wind and temperature profiles, the critical amplitude depends on the wavelength, which is greatest for a wavelength of 5-10 km and decreases both for longer waves and shorter waves. The occurrence of turbulent spots, which are disseminated in a stably stratified flow, on the crests of internal waves as the result of "secondary instability" is thus most probable in the inversion layers and in general in layers having a temperature gradient which changes with altitude, and also in layers with large vertical shifts of the wind. Such zones must have comparatively small dimensions (several kilometers along the horizontal); these dimensions are smaller, the greater is the stability of stratification in the layer. The formation of discontinuous turbulent zones is very probable (on the crests of several of the largest waves in the group). The occurrence of turbulent zones of large dimensions (several tens and hundreds of kilometers) may be naturally related to the "primary" instability of the main flow. / 342

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